Mapping Teacher Moves when Facilitating Mathematical Modelling

Jill P Brown
Deakin University
jill.brown@deakin.edu.au

Gloria A Stillman
Retired
stillmanga@gmail.com

This paper explores use of a set of diagrammatic tools for representation and analysis of the moves a teacher makes implementing a mathematical modelling task. The focus here is on identifying what the teacher did so we can subsequently interrogate this, as to the why. Data include pre and post lesson teacher interviews and transcripts of a video and audio-recorded task implementation. The analytical tools developed, with one teacher and one task early in a three-year project were particularly useful in ascertaining what the teacher moves were as we subsequently sought to determine reasons for these.

Our focus is on why teachers do what they do, particularly when implementing mathematical modelling tasks with upper secondary students. The first step in answering the 'why' question is to ascertain 'what' they do. The research reported here is part of the Victorian component of the Australian Research Council *Discovery Project* (DP17010555), *Using Mathematics to Solve Real-World Problems: The Role of Enablers Project*. This two-state research project, Queensland and Victoria, included a focus on identifying teaching approaches supportive of Year 10 and 11 students' modelling. We focused on identifying enablers of student success during mathematical modelling (Geiger et al. 2018). The two states have distinct characteristics with regard to expectations for mathematical modelling (Brown, 2020).

The Victorian Curriculum and Assessment Authority (VCAA) Study Design: Mathematics in the Victorian Certificate of Education (VCE) describes essential mathematical activities as including "conjecturing, hypothesising and *problem posing*; estimating, calculating, and computing; abstracting, proving, refuting and inferring; applying, investigating, *modelling and problem solving*" (emphasis added, VCAA, 2015, p. 6). Our interest is firmly centred on mathematical modelling and the associated problem posing and problem solving. Mathematical modelling is currently an expected part of teaching, learning, and assessment in the Victorian upper secondary curricula (i.e., the VCE). With regards to assessment, modelling tasks are expected to be of 2-3 hours duration. In Year 10, modelling is described within the *Learning in Mathematics* statement as part of the problem-solving proficiency, with students expected to consider both unfamiliar and meaningful problems and in doing so "make choices, interpret, formulate, model and investigate problem situations, select and use technological functions and communicate solutions effectively" (VCAA, 2016). Adding to the complexity of the classroom, but also the complexity and authenticity of accessible real-world contexts, students are also expected to make use of mathematical digital technology. In VCE Mathematics, one of the three aims is for students to "use technology effectively as a tool for working mathematically" (VCAA, 2015, p. 6) with students engaging with contexts and problems from "well defined and familiar to open ended and unfamiliar" (VCAA, 2016, p. 6). This can increase student engagement (Stillman et al., 2010), be used throughout the modelling process (Stillman et al. 2015), but further research is needed (Stillman & Brown, 2014).

However, Wedelin and Adawi (2015, p. 26) suggest that school students are generally "taught to *solve well-defined problems by using given methods*". Thus, students too often have scant experiences with the early stages of problem solving which is unavoidable in modelling where students need to make sense of the messy real-world situation and subsequently formulate a problem (i.e., mathematise) that can be solved mathematically. For example, 'best' will need to be defined by the task solvers in many modelling tasks. Attempts to solve modelling tasks typically involves struggle—the task can be challenging to understand as well as challenging to solve. Different solvers may make a different determination of best which can be a novel situation for learners who have mainly experienced one way to solve a problem according to Wedelin and Adawi (p. 30).

(2023). In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives. Proceedings of the 45th annual conference of the Mathematics Education Research Group of Australasia* (pp. 115–122). Newcastle: MERGA.

The nature of the inherent complexity and hence challenge in solving modelling tasks generally sees students working together to solve tasks. This aligns with the argument of Stillman et al. (2020) that the practices in school mathematics classrooms should mirror those of professional mathematical modellers. In particular, the practices of making meaning, anticipating or engaging in mathematical foresight, arguing mathematically, and validating or justifying. These practices are inherent in mathematical modelling at school level when students work in co-present groups (Hager & Becket, 2019). Classroom discourse has the productive effect of "making thinking visible" (Collins et al., 1991, p. 38). When students articulate their thinking, not only is the group more likely to make progress, but also teachers might be more likely to withhold or delay in-the-moment scaffolding as they are more aware of student reasoning. Teachers are the key to developing students as successful modellers, including in articulating their thinking within their group, and to the class. Once modelling is included in curriculum documents, *whether* it is implemented and *how* it is implemented depends on the teacher (Chapman, 2007). Our focus, within a context explicitly including mathematical modelling, is, what is the thinking of teachers who include modelling in their teaching and what pedagogical strategies do they use that facilitate modelling?

Background to the Study

To illustrate the complexity involved in mathematical modelling we include the modelling cycle diagram (see Figure 1) (itself a model) used by researchers, teachers, and students. This modelling cycle diagram has been particularly useful both in developing a shared understanding between researchers and teachers of the complexities involved in modelling but also as a useful scaffold by teachers and students during modelling. Seven stages, A-G, are shown beginning in the messy real world. During modelling, modellers move back and forward between stages, rather than as simplified in the diagram. Between each stage, represented by an arrow, are sets of activities required to transition from one stage to another. Examples of relevant activities are part of the diagram. Between the stages, are double headed arrows representing metacognitive activity, an essential part of successful modelling.

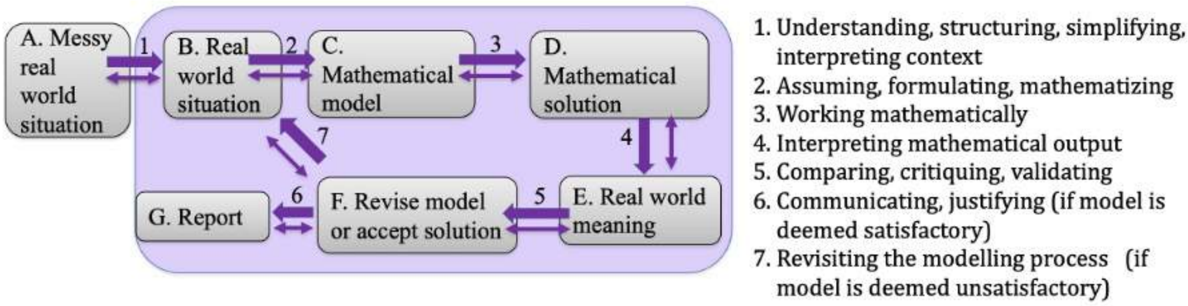

Figure 1. Representation of modelling cycle diagram used in the project (Stillman, 2011).

Elsewhere we have focused on student reasoning in data-rich modelling tasks (Stillman & Brown, 2023) and teacher tactics (Brown & Stillman, 2023). In this paper we focus on the actions of the teacher. We are particularly interested in ascertaining *why teachers do what they do*, but this must be preceded by determining *what teachers do*. In any mathematics classroom, this can be a challenge. It is certainly a challenge in upper secondary classrooms with students working in groups using digital technology and working on challenging modelling tasks.

Research Design and Methodology

One teacher is the focus of the analysis here, selected based on her participation in year 1 of the project. As an experienced teacher of mathematics, including mathematical modelling, we expected she would be able to articulate the reasons for her actions as well as if these were planned or based on in-the-moment decisions. The school was a regional senior secondary college with co-educational classes. The Year 11 year-long mathematics subject included expectations of problem solving, investigations, and modelling (VCAA, 2015). Due to administrative challenges in the research project, the class was only able to work on one modelling task, *Bush Walking with Hilary* (BWwH). In seeking to determine what the teacher was doing and saying, as she interacted with the co-present groups (Hager & Becket, 2019)—our unit of analysis—we employed multiple data sources and several analytical techniques. Although this paper focusses on one teacher, our close analysis of the data was intended to apply to other components in the three-year project (different teachers, classes, tasks, and over time). Data included in the analysis reported here includes semi-structured pre- and post-lesson implementation teacher interviews (Flick, 2006), video and audio recordings, and field notes.

Using video and audio recordings, student scripts, and researcher field notes, we created a case record (Miles et al., 2014) for individual student groups. The case record included the transcript, which documented the discourse and actions of the students and teacher. These were supplemented with stills from the video recordings, at times annotated to show gestures of interest. Subsequently a case record for the teacher was constructed. What occurred as the lesson unfolded may not be so obvious to the researcher. For example, a teacher may spend more time interacting with one group than others. This may have been pre-planned. For example, a group included a student new to the class, so the teacher wanted to ascertain the group collaboration was proceeding smoothly (i.e., teacher intends to interact closely with Group 1). The reason may be more general with the intention focused on working with any group apparently struggling to get started (i.e., teacher interacts with Group 2 who seem to be unfamiliar with the context) (see Stillman & Brown, 2023). The interactions, intended or realised, may be of increased frequency or of longer duration. However, as we had multiple data sources, including pre and post teacher interviews, we had rich data to interrogate.

Teacher Moves

Diagrams are particularly useful analytical tools (Miles et al., 2014). Diagrams show what is hidden within transcripts and case records. Diagrams also allow us to focus simultaneously rather than sequentially on patterns in the data (p. 108). One question of interest related to the way in which teacher interactions with students unfolded during task implementations. To analyse activity in the classroom as students collaborated on a modelling task, we coded the case record into four categories of interactions. We refer to *Teacher Group Interactions* (TGIs) as occurring when the teacher interacted with a student group. The TGI could be initiated by the teacher or a student. Unsurprisingly, at times an interaction was preceded by the teacher listening to what a particular group was saying or doing. At times the teacher engaged in this listening but did not immediately follow this with a TGI. We described such interactions as *Teacher Group Listening* (TGL). The student group may or may not have been aware of a TGL. Where the TGL was immediately followed by an interaction, this was coded as part of the TGI. On some occasions, this also occurred at the conclusion of an interaction. *Whole Class Interactions* (WCI) occurred when the teacher was interacting with the whole class. We note that as the students are upper secondary students, at times they chose to ignore the teacher's intentions. We coded *Group Interactions* (GI) where the students were working on the task, independently of the teacher. At times this involved some or all group members in discourse with each other, sharing screens from their technological devices, and working together to record ideas and outputs from technological devices. At other times, individuals in the group were working more independently, albeit mainly toward a common goal.

Sequence Diagrams

In looking at how the teacher moved around the room, we asked how the interactions with each group unfolded. There were seven student groups working on the *BWwH task.* Figure 2 shows that she interacted with each group, before a second interaction with any group occurred, with the exception of Group 6. The first interaction was with Group 7, followed by Group 6 and Group 4. The shaded rectangle indicates a TGL. This analysis and subsequent diagram told some, but not all, of the story of how the teacher interacted with the students.

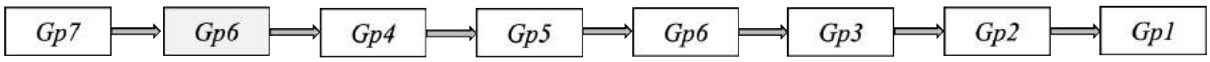

Figure 2. Sequence whereby TB1 interacts with all groups.

Mud Maps

The layout of the room may also have influenced the order of the TGIs so we created a mud map of the classroom layout, as shown in Figure 3. A mud map is a rough sketch (later digitised) created by the researchers at the beginning of the lesson. Each rectangle represents the table and approximate location of the named group. Group 1 is located at the front of the room adjacent to the teacher table, whereas Group 6 was located at the back to the room near the door. Several video and audio recorders were located in the room to capture as much of the rich data as possible, but these are not shown on in Figure 3. On the righthand side of Figure 3, the teacher's initial set of interactions with the student groups staring from the yellow dot is shown. The mud map represents the same information as Figure 1, but this two-dimensional view provides a greater sense of how these interactions unfolded in the actual classroom.

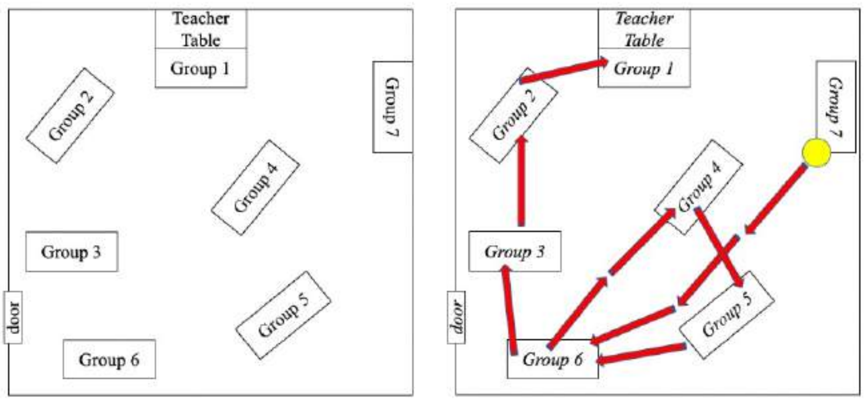

Figure 3. Mud map of the classroom setup by the teacher (left) and the initial teacher group interactions.

The layout of the room was considered to determine if this might have influenced the sequence of interactions. In some lessons in the project, the teacher deliberatively set up the room, in others, the furniture was used as left by the previous class. How the groups were determined, and located in the room, also varied (teacher or student selection or a combination) and these were considered in our analysis. In this class, the teacher selected the groups of two to three students and indicated where they should sit during the modelling lesson.

Timeline Diagrams

Noting that the sequence diagram showing teacher student interactions highlighted some aspects of what occurred, we also created timeline diagrams. The timeline is shown in 6 second segments and we acknowledge times are estimates, due to rounding and also using multiple recording devices that began recording at different times. Our careful re-listening and re-watching the recordings has however resulted in an accurate portrayal of what occurred in the classroom. The timeline diagram has three sets of columns. Column 1 is segmented in 30 second sections, white for the first 30 seconds, then grey for the second thirty seconds of each minute. Each of these are further partitioned into five 6 second sections in column 2. Column 3 indicates the type of interactions. A section of a timeline diagram is presented in Figure 4.

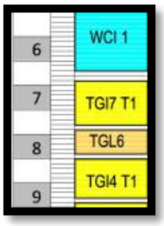

Figure 4. A snippet of a timeline diagram for the class.

Initially, a WCI is in progress as the teacher introduced the task and initiates a discussion about the context. This is followed by 12 seconds where the teacher is not interacting with the students. From 6:12-6:18 mins the first TGI occurs as the teacher initiated an interaction with Group 7. The coding T1 indicates this is the first teacher-initiated interaction with this group. This is followed by the teacher listening to, but not interacting with Group 6. Next, we see the first interaction between the teacher and Group 4, also initiated by the teacher. The white rectangles in column 3 are indicative of the times when the teacher might be moving from one group to another, or watching the class more globally, or preparing to bring the class together for a WCI. The case record would be examined where this detail was needed.

Analysis and Findings

Our analysis, although presented here for the *Bush Walking with Hilary* (BWwH) implementation, is focused on what we can learn about 'why teachers do what they do' more broadly. The sequence of teacher-student interactions over the course of the lesson is presented in Figure 5 with the mud map of these in Figure 6 (with arrows coloured red, orange, green then black). Each new row shows where the teacher began a new mini sequence of interactions. In rows 1 and 2, a TGI occurred with each group, indicating interaction with every group. However, in row 3 only five TGIs occurred before returning to Group 6. This part of the lesson saw no interactions with Groups 4 and 5. This sequence suggests the teacher's intention was to interact with each group and then repeat this as the lesson unfolded. The location of WCI2-4 in the sequence is indicated by black arrows with WCI1 and WCI5 bookending the sequence.

The timeline diagram (Figure 7) indicates that in this modelling lesson implementation, the teacher began with an extended *Whole Class Interaction* (WCI1, 6 mins 13 secs) and most of the lesson saw the students working in their groups. Three shorter WCIs occurred in the final stages of the lesson and a final WCI occurred as the teacher wrapped up the lesson. The timeline diagram also shows the teacher interacting with various groups for almost all of the lesson. The number of TGIs per group varied in number (3-5) and duration (13 sec—5:13 mins). The shortest TGI (TGI6 T1) came shortly after the only TGL. The total TGI time per group varied from 1:30 minutes (Group 1) to 9:29 mins (Group 6).

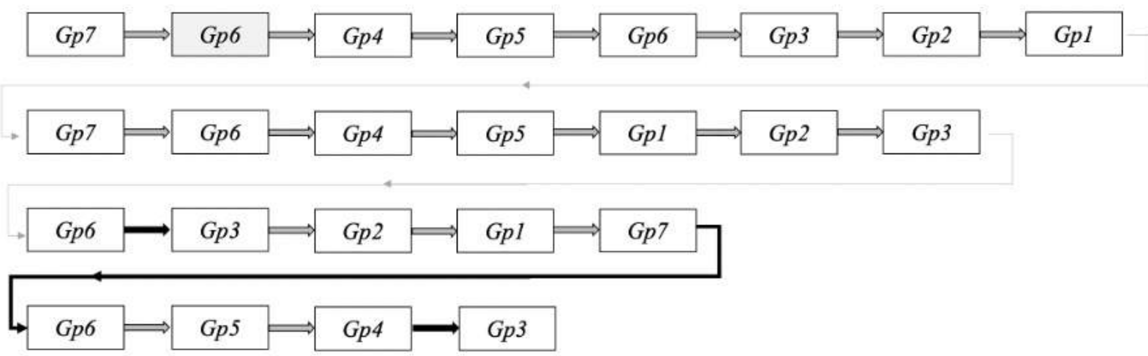

Figure 5. Sequence of teacher-student interactions during task implementation.

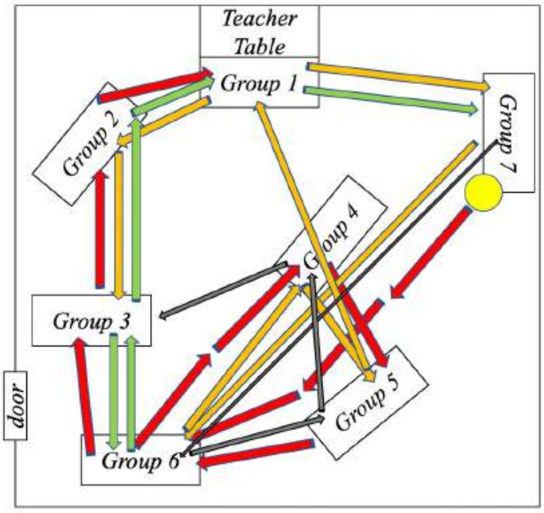

Figure 6. The complete mud map with TGIs and TGL for the class undertaking the *BWwH task*.

Discussion and Concluding Remarks

While our ultimate focus in on *why teachers do what they do,* this paper focuses on ways of ascertaining *what they do*. The diagrams proved particularly useful in guiding further analysis of the complexity of the thinking (Hager & Beckett, 2019) both mathematical and pedagogical. Discourse analysis was very much a focus of our broader analysis (e.g., Stillman & Brown, 2023) during various types of interaction and the visual displays created and presented here were particularly productive. Here we report our broad categories of discourse in this task implementation as the teacher interacted with the co-present groups. For this teacher, we generalise three distinct categories of interactions, based on what the teacher did and said. These correspond with Row1, 2, and the combined rows 3 and 4 presented in Figure 5 and coloured paths in Figure 6. In the first set of TGIs, the teacher asked each group to respond to her question: *what is the problem about?* In her second mini sequence of teacher student interactions, she again asked, *what is the problem?* but added, *what is your approach*? In the third mini sequence, her focus was: *can you verify your model? can you convince me it works?* During the WCIs, she encouraged students to *write down where you are at*.

In her pre-lesson implementation interview, the teacher explained she thought the students would be "be very slow to get started. This particular class like rules, formulae, here's how you do it, go and apply these rules. We don't do a lot of open-ended questions." She elaborated further, "slow to start and then they are very talkative, … they'll follow each other a little bit".

She wanted to see "if they adapt and change [their] planning if it's not working … keep the good bit …. syphoning out what bit is the wrong bit, that's not helping and throwing that away but keeping the part" that is on track. After explaining how she would introduce the task, she described her approach of supporting students, "I won't say do this, do that, I'll give them hints to think about" noting she would give more structured hints if "there's a group that's not really able to get started" [Pre-lesson implementation interview].

As the lesson unfolded, it was evident that one purpose of the teacher's interactions was to encourage collaboration and communication even if she did not always, in the moment, follow what they were doing. "They were a little bit hesitant to say what they thought they were doing. They didn't communicate it very well" [Post-lesson implementation interview]. "When they were telling me what they were doing, sometimes I knew what they were doing and made out I didn't, so they'd say it a bit better" thus helping them communicate their thinking. At other times, "I really didn't know what they were saying, they didn't make sense."

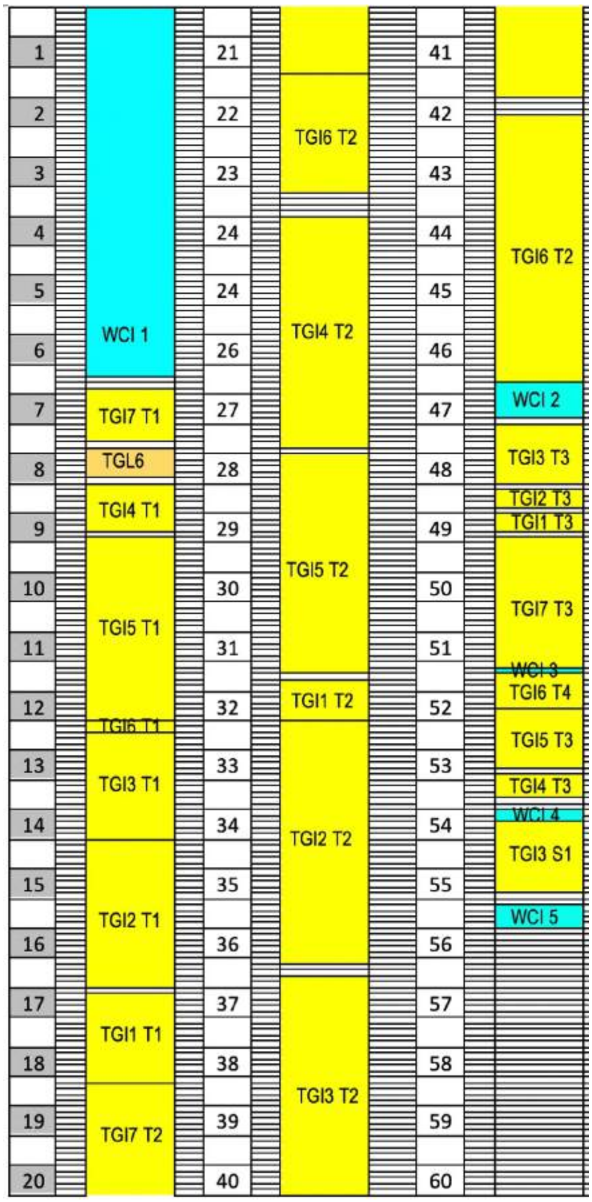

Figure 7. The complete timeline diagram for the class undertaking the *BWwH task*.

The analysis presented here, particularly the various diagrams, was particularly useful in helping us determine what the teacher did during task implementation. This subsequently guided the focus of our analysis as to why she acted in particular ways. The methods used here were then applied to other classrooms in our study, looking for commonalities and variation within and across classes. Each type of diagram, sequence, mud map, and timeline, contributed both individually and collectively to our understanding of *what the teacher did*. Each provides different insight as to what the teacher does. They provide different lenses for us as we also focus on the discourse to further consider *why teachers did what they did*. Our diagrams are handcrafted, and researcher generated, but powerful in allowing us to 'see' what the actors did. We concur with Miles et al. (2014, p. 108), that visual displays presenting information systematically allow "credible and trustworthy analysis" to answer our research questions. Noticing continues to be crucial to successful modelling (Galbraith et al. 2017).

Acknowledgments

DP17010555 Chief Investigators were V. Geiger, G. Stillman, J. Brown, and P. Galbraith. M. Niss was a Partner Investigator. The views expressed herein are those of the authors.

References

Brown, J. P. (2020). Expectations for challenge in modelling and its assessment In G. A. Stillman, G. Kaiser, & C. E. Lampen (Eds.), *Mathematical modelling education and sense-making* (pp. 371–382). Springer. https://doi.org/10.1007/978-3-030-37673-4_32

Brown, J., & Stillman, G. (2023). *Teaching Tactics to Manage Mathematisation during Mathematical Modelling,* [Manuscript submitted for publication.]

Chapman, O. (2007). Mathematical modelling in high school mathematics: Teachers' thinking and practice. In W. Blum, P. L., Galbraith, H. W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education*. Springer. https://doi.org/10.1007/978-0-387-29822-1_34

Collins, A., Brown, J. S., & Holum, A. (1991). Cognitive apprenticeship: Making thinking visible. *American Educator, 6*(11), 38–46.

Flick, U. (2006). *An introduction to qualitative research* (3rd ed.). Sage.

Galbraith, P., Stillman, G., & Brown, J. (2017). The primary of 'noticing': A key to successful modelling. In G. Stillman, W. Blum, & G. Kaiser (Eds.), *Mathematical modelling and application* (pp. 83–94). Springer.

Geiger, V., Stillman, G., Brown, J., Galbraith, P., & Niss, M. (2018). Using mathematics to solve real world problems: The role of enablers*. Mathematics Education Research Journal*, *30*(1), 7–19.

Hager, P., & Beckett, D. (2019). *The emergence of complexity: Rethinking education as a social science*. Springer.

Miles, M., Huberman, M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). Sage.

Stillman, G. (2011). Applying metacognitive knowledge and strategies in applications and modelling tasks at secondary school. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 165–180). Springer.

Stillman, G., & Brown, J. (2014). Evidence of "implemented anticipation" in mathematising by beginning modellers. *Mathematics Education Research Journal 26*(4), 763–789.

Stillman, G. A., & Brown, J. P. (2023). Modeling the phenomenon versus modeling the data set. *Mathematical Thinking and Learning*, *25*(3). https://doi.org/10.1080/10986065.2021.2013144

Stillman, G. A., Brown, J. P., & Czocher, J. (2020). Yes, mathematicians do X so students should do X, but it's not the X you think. *ZDM*, *52*(6), 1211–1222. https://doi.org/10.1007/s11858-020-01183-5

Stillman, G., Brown, J., & Galbraith, P. (2010). Identifying challenges within transition phases of mathematical modeling activities at year 9. In R Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 385–398). Springer.

Stillman, G., Brown, J., & Geiger, V. (2015). Facilitating mathematisation in modelling by beginning modellers in secondary schools. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modeling in education, research and practice: Cultural, social, and cognitive influences* (pp. 93–103). Springer.

VCAA. (2015). *Victorian Certificate of Education: Mathematics study design* (2016–2020). VCAA.

VCAA. (2016, online). *Victorian Curriculum Mathematics: Foundation-10*. VCAA.

Wedelin, D., & Adawi, T. (2015). Warming up for PBL: A course in mathematical modelling and problem solving for engineers. *Högre utbildning, 5*(1), 23–34.